Bayesian Networks

1) Consider the following Bayesian network, where F = having the flu and C = coughing:

P(F) = 0.1   (F) $\longrightarrow$ (C)   P(C | F) = 0.8
                                                P(C | ¬ F) = 0.3
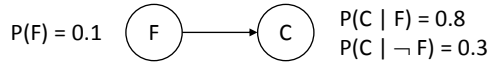
a) Write down the joint probability table specified by the Bayesian network.

b) Determine the probabilities for the following Bayesian network

(C) $\longrightarrow$ (F)
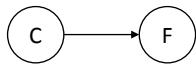
so that it specifies the same joint probabilities as the given one.

c) Which Bayesian network would you have specified using the rules learned in class?

d) Are C and F independent in the given Bayesian network?

e) Are C and F independent in the Bayesian network from Question b?

2) To safeguard your house, you recently installed two different alarm systems by two different reputable manufacturers that use completely different sensors for their alarm systems.

a) Which one of the two Bayesian networks given below makes independence assumptions that are not true? Explain all of your reasoning. Alarm1 means that the first alarm system rings, Alarm2 means that the second alarm system rings, and Burglary means that a burglary is in progress.

(1) (Burglary)   (2) (Alarm1)   (Alarm2)

(Alarm1)   (Alarm2)   (Burglary)
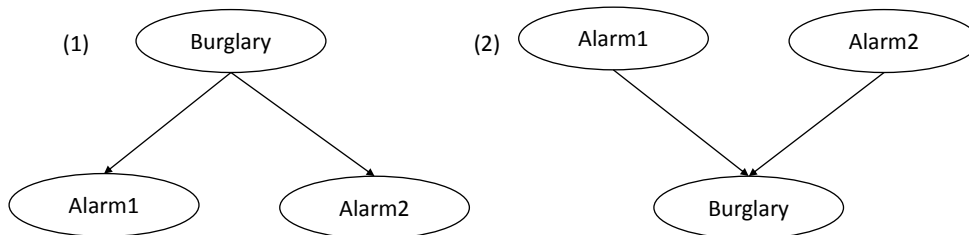
b) Consider the first Bayesian network. How many probabilities need to be specified for its conditional probability tables? How many probabilities would need to be given if the same joint probability distribution were specified in a joint probability table?

c) Consider the second Bayesian network. Assume that:

P(Alarm1) = 0.1
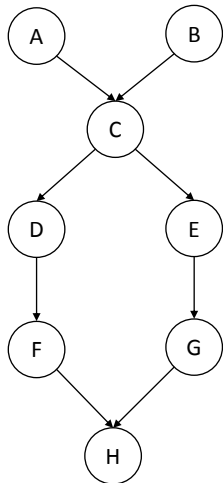
P(Alarm2) = 0.2

P(Burglary | Alarm1, Alarm2) = 0.8

$P(\text{Burglary} \mid \text{Alarm1}, \neg \text{Alarm2}) = 0.7$

$P(\text{Burglary} \mid \neg \text{Alarm1}, \text{Alarm2}) = 0.6$

$P(\text{Burglary} \mid \neg \text{Alarm1}, \neg \text{Alarm2}) = 0.5$

Calculate $P(\text{Alarm2} \mid \text{Burglary}, \text{Alarm1})$. Show all of your reasoning.

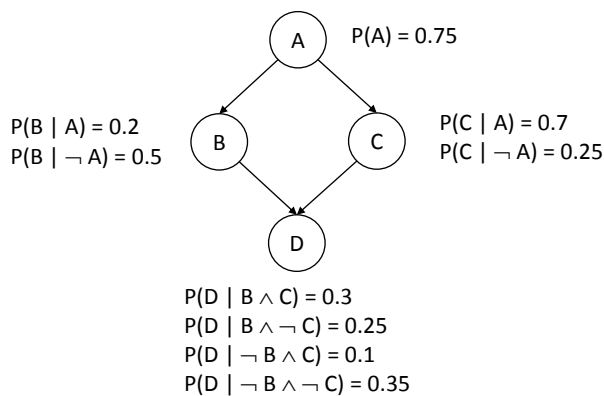**3)** Consider the following Bayesian network:

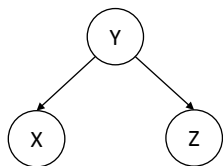a) Are D and E necessarily independent given evidence about both A and B?

b) Are A and C necessarily independent given evidence about D?

c) Are A and H necessarily independent given evidence about C?

4) Consider the following Bayesian network. A, B, C, and D are Boolean random variables. If we know that A is true, what is the probability of D being true?

$P(A) = 0.75$

$P(B \mid A) = 0.2$
$P(B \mid \neg A) = 0.5$

$P(C \mid A) = 0.7$
$P(C \mid \neg A) = 0.25$

$P(D \mid B \wedge C) = 0.3$
$P(D \mid B \wedge \neg C) = 0.25$
$P(D \mid \neg B \wedge C) = 0.1$
$P(D \mid \neg B \wedge \neg C) = 0.35$

5) For the following Bayesian network

we know that X and Z are not guaranteed to be independent if the value of Y is unknown. This means that, depending on the probabilities, X and Z can be independent or dependent if the value of Y is unknown. Construct probabilities where X and Z are independent if the value of Y is unknown, and show that they are indeed independent.