First-Order Logic

1) Translate the following English sentences to first-order logic using the following predicates: Owns(x, y), Dog(x), Cat(x), Cute(x), and Scary(x). For example, Owns(x, y) means that object x owns object y:

(a) Joe has a cute dog.

(b) All of Joe's dogs are cute.

(c) Unless Joe owns a dog, he is scary.

(d) Either Joe has at least one cat and at least one dog or he is scary (but not both at the same time).

(e) Not all dogs are both scary and cute.

2) Translate the following sentences in first-order logic to English. Apple(x) means that object x is an apple, Red(x) means that object s is red, Loves(x, y) means that person x loves person y:

(a) $\forall x$ (Apple$(x) \Rightarrow$Red(x))

(b) $\forall x \; \exists y$ Loves(x, y)

(c) $\exists y \; \forall x$ Loves(x, y)

3) Specify what a grandmother is, using the predicates IsGrandMotherOf, IsMotherOf and IsFatherOf. IsGrandMotherOf(x, y) means that person x is the grandmother of person y, IsMotherOf(x, y) means that person x is the mother of person y, and IsFatherOf(x, y) means that person x is the father of person y. Define additional predicates if needed.

4) For each of the following sentences in first-order logic, specify whether it is valid, satisfiable, and/or unsatisfiable:

(a) $P(A) \Rightarrow \forall x P(x)$

(b) $P(A) \Rightarrow \forall x \neg P(x)$

(c) $P(A) \Rightarrow \exists x P(x)$

(d) $P(A) \Rightarrow \exists x \neg P(x)$

5) Solve Problem 9.23 on page 365 of our textbook.