Propositional Logic

1) Translate the following Propositional Logic to English sentences.
Let:

- E=Liron is eating
- H=Liron is hungry

(a) $E \Rightarrow \neg H$

(b) $E \wedge \neg H$

(c) $\neg(H \Rightarrow \neg E)$

2) Translate the following English sentences to Propositional Logic.
Propositions: (R)aining, Liron is (S)ick, Liron is (H)ungry, Liron is (HA)appy, Liron owns a (C)at, Liron owns a (D)og

(a) It is raining if and only if Liron is sick

(b) If Liron is sick then it is raining, and vice versa

(c) It is raining is equivalent to Liron is sick

(d) Liron is hungry but happy

(e) Liron either owns a cat or a dog

3) Which of the following propositions are tautologies? Which are contradictions? Why?

(a) Three is a prime number.

(b) It is raining or it is not raining.

(c) It is raining (P) and it is not raining ($\neg P$).

4) Which of the following propositions are tautologies? Why?

(a) P

(b) $P \Rightarrow P$

(c) $(P \Rightarrow P) \Rightarrow P$

(d) $P \Rightarrow (P \Rightarrow P)$

5) Which of the two following propositions are equivalent in the sense that one can always be substituted for the other one in any proposition without changing its truth value? Why?

(a) first proposition:$P \Rightarrow Q$ second proposition: $\neg P \vee Q$

(b) first proposition: $\neg P$ second proposition: $P \Rightarrow False$

(c) first proposition: $\neg P$ second proposition: $False \Rightarrow P$

(d) first proposition: $\neg P$ second proposition: $\neg P \vee Q$

6) Is it possible that

(a) $(KB \models S)$ and $(\neg KB \models S)$

(b) $(KB \models S)$ and $(KB \models \neg S)$

(c) $(KB \models S)$ and $(KB \not\models S)$

(d) $(KB \models S)$ and $(KB \not\models \neg S)$

(e) $(KB \not\models S)$ and $(KB \not\models \neg S)$

(f) $(KB \not\models S)$ and $(\neg KB \not\models S)$

If so, provide an example. If not, explain why it is impossible.

7) Prove that $P \wedge Q \models P \vee Q$.

8) Consider the following popular puzzle. When asked for the ages of her three children, Mrs. Baker says that Alice is her youngest child if Bill is not her youngest child, and that Alice is not her youngest child if Carl is not her youngest child. Write down a knowledge base that describes this riddle and the necessary background knowledge that only one of the three children can be her youngest child. Show with resolution that Bill is her youngest child.

9) Consider the following popular puzzle. A boy and a girl are talking. "I am a boy" said the child with black hair. "I am a girl" said the child with white hair. At least one of them is lying. Write down a knowledge base that describes this riddle. Show with resolution that both of them are lying.

10) In the back of a magazine you find a riddle: "Suppose that liars always speak what is false, and truth-tellers always speak what is true. Further suppose that Amy is either a liar or a truth-teller." The riddle then provides some additional facts about Amy and asks whether Amy has to be a truth-teller. Excitedly, you encode the facts in propositional logic and implement a resolution procedure on your computer. Since you do not make any mistakes, the computer will give you the correct answer. You ask the computer whether the facts entail that Amy is a truth-teller.

a) The computer tells you that the facts entail that Amy is a truth-teller. Since the text states that Amy is either a liar or a truth-teller, can you conclude that Amy is not a liar?

b) The computer tells you that the facts do not entail that Amy is a truth-teller. Since the text states that Amy is either a liar or a truth-teller, can you conclude that Amy is a liar?