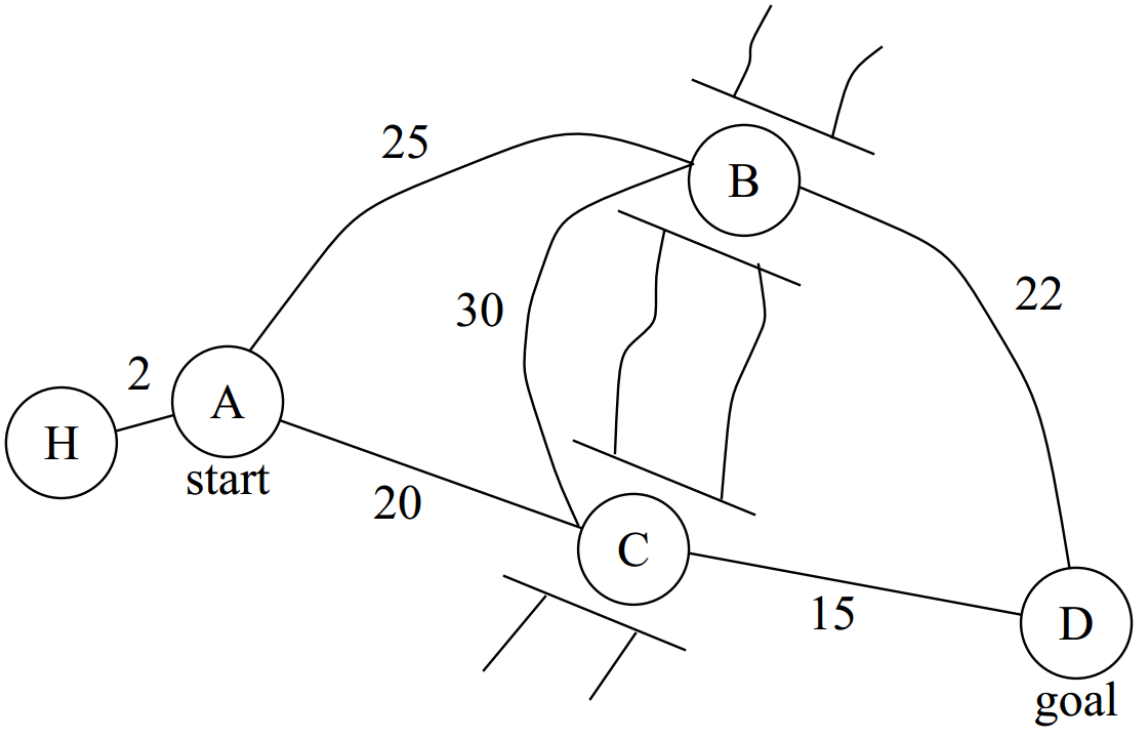
Decision Theory– Solution

1) A mobile robot is trying to get from his current position A to a destination D as quickly as possible. There is a river separating A from D and there are two bridges, B and C, spanning the river. The robot must design a strategy to move from A to D via one of the bridges.



Though the robot knows that one (and only one) of the bridges is inoperable, it is uncertain regarding which one of the two bridges is out. From its start position, position A, the robot can climb a hill to position H and use sensors to obtain information regarding which bridge is out. Collecting this information will take time, since it must go to H and return back to A, and the information gained is uncertain because the sensors will not be able to tell precisely which bridge is out.

The numbers in the above graph give the distance between locations in miles. In similar situations in the past, the robot experienced that 4 out of 5 times bridge C was out and only 1 out of 5 times bridge B was out. The robot has a short-range sensor that tells it with 100 percent reliability whether a bridge is out. The sensor can only be used when the robot is directly in front of the bridge. The long-range sensor of the robot is unreliable. It errs with a probability of 10 percent, that is, suggests that the broken bridge is operable and the other bridge is broken.

Design a strategy for the robot that minimizes the expected execution time.

Answer:
The plan that minimizes the expected execution time is the following: the robot should go from A to H and use its long-range sensor. If the long-range sensor reports that bridge B is operational, the robot should then go to B via A and, if the bridge is indeed operational, continue to D. If the bridge is not operational, the robot has to continue to D via C. If the long-range sensor reports that bridge B is not operational, the robot should go to C via A and, if the bridge is indeed operational, continue to D. If the bridge is not operational, the robot has to continue to D via B. The expected execution time of this conditional plan is 51.3 minutes. The value of information of climbing the hill is 51.6 - 51.3 + 4.0 = 4.3 minutes.

The calculations are as follows:

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B = bridge B is operable (and bridge C is broken)
C = bridge C is operable (and bridge B is broken)
b = long range sensor reports that bridge B is operable (and bridge
C is broken)
c = long range sensor reports that bridge C is operable (and bridge
B is broken)

P(B) = P(NOT C) = 4/5 = 0.8000
P(NOT B) = P(C) = 1/5 = 0.2000
P(NOT b | B) = P(c | B) = 0.1000
P(b | B) = P(NOT c | B) = P(b | NOT C) = P(NOT c | NOT C) = 0.9000
P(b | NOT B) = P(NOT c | NOT B) = P(b | C) = P(NOT c | C) = 0.1000
P(NOT b | NOT B) = P(c | NOT B) = P(NOT b | C) = P(c | C) = 0.9000
P(NOT b | B) = P(c | B) = P(NOT b | NOT C) = P(c | NOT C) = 0.1000
P(b) = P(b AND B) + P(b AND NOT B)
     = P(b | B) P(B) + P(b | NOT B) P(NOT B) = 0.7400
P(NOT b) = 1 - P(b) = 0.2600
P(B | b) = P(b | B) P(B) / P(b) = 0.9730
P(NOT B | b) = 1 - P(B | b) = 0.0270
P(B | NOT b) = P(NOT b | B) P(B) / P(NOT b) = 0.3077
P(NOT B | NOT b) = 1 - P(B | NOT b) = 0.6923
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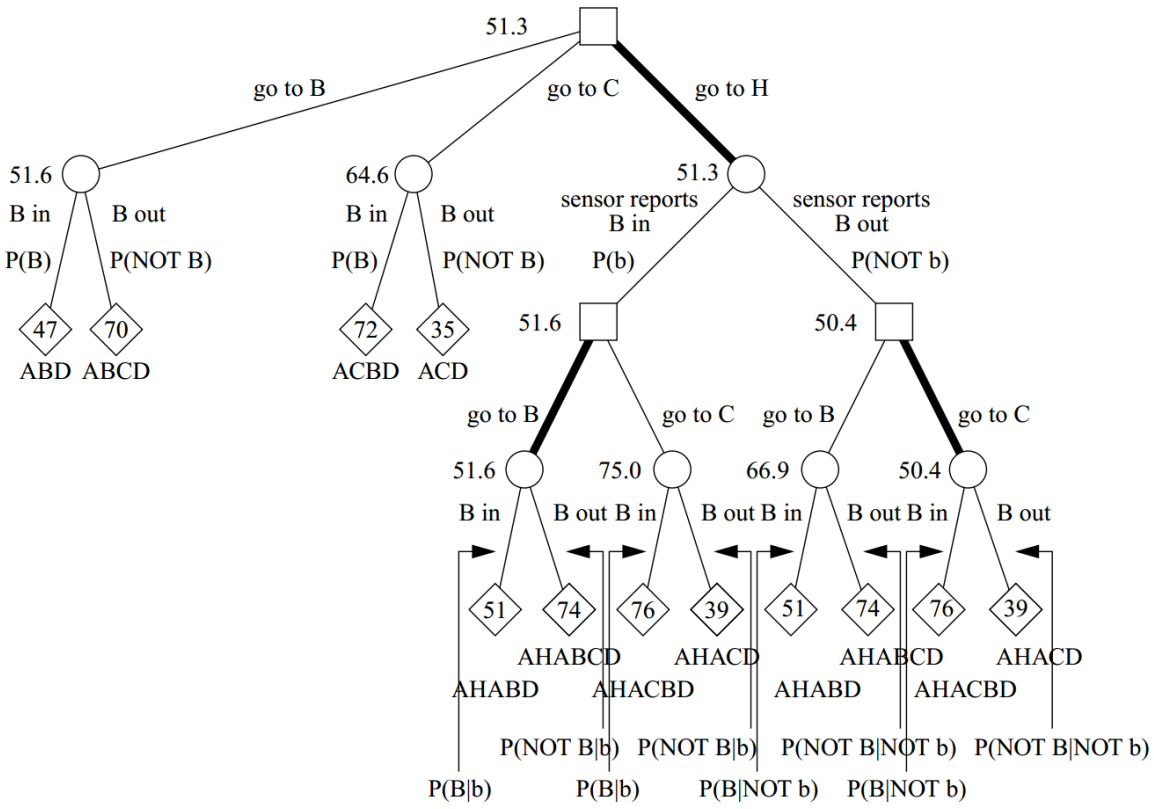
51.3
go to B
go to C
go to H
51.6
B in
B out
P(B)
P(NOT B)
47
70
ABD
ABCD
64.6
B in
B out
P(B)
P(NOT B)
72
35
ACBD
ACD
51.3
sensor reports
B in
sensor reports
B out
P(b)
P(NOT b)
51.6
50.4
go to B
go to C
go to B
go to C
51.6
75.0
66.9
50.4
B in
B out B in
B out B in
B out B in
B out
51
74
76
39
51
74
76
39
AHABCD
AHACD
AHABCD
AHACD
AHABD
AHACBD
AHABD
AHACBD
P(NOT B|b)
P(NOT B|b)
P(NOT B|NOT b)
P(NOT B|NOT b)
P(B|b)
P(B|b)
P(B|NOT b)
P(B|NOT b)