# Decision Theory

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Russell and Norvig, 3rd Edition, Sections 16.1-16.3 and 16.5

These slides are new and can contain mistakes and typos.
Please report them to Sven (skoenig@usc.edu).

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## Decision Theory

- We now start with probabilistic planning by studying “decision trees.”
- They are similar to decision networks from decision theory.
- They are also similar to game trees from game playing.
- They are completely different from decision trees in machine learning.

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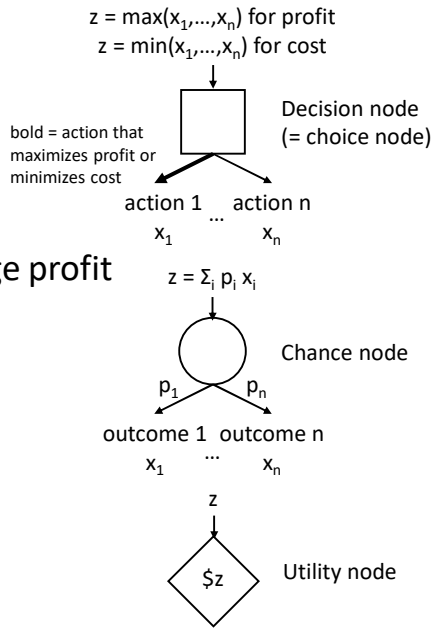
# "Decision Trees"

- We are playing a game against nature.
- Decision (or choice) nodes:
  We pick the action that maximizes our average profit (or minimizes our average cost).
- Chance nodes:
  Nature picks outcomes randomly according to given probabilities.
- Utility nodes:
  We make the given total profit (or incur the given total cost).

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# Example: "Decision Trees"

- An oil company can buy **one** of 4 indistinguishable blocks of ocean drilling rights, exactly one of which contains oil worth 5 (million) dollars. The price of each block is 1 (million) dollars. Should it buy one of the blocks or should it abstain from the purchase to maximize its expected profit?

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# Example: "Decision Trees"

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- Human decision makers are often risk-averse in high-stake decision situations that do not repeat, that is, they consider more the worst-case outcomes than the average outcomes. Here, however, we assume that they consider only the average outcomes and thus maximize their expected profit (or minimize their expected cost).

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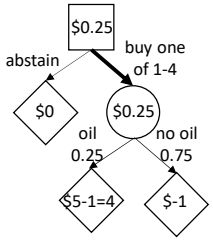
# Example: "Decision Trees"

- An oil company can buy one of 4 indistinguishable blocks of ocean drilling rights (numbered 1-4), exactly one of which contains oil worth 5 (million) dollars. The price of each block is 1 (million) dollars. Should it buy one of the blocks or should it abstain from the purchase?

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## Example: “Decision Trees”

- Answer: It should buy one of the blocks for an expected profit of 0.25 (million) dollars.
- The plan is: Buy one of the blocks.

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## Example: “Decision Trees”

- In addition, a seismologist offers the company the results of a survey of Block 1, which indicates definitively whether the block contains oil, for 1 (million) dollars. Should the company buy the information?

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Here, the various probabilities are easy to calculate.
However, often you will have to use what you learned about probability theory to calculate them.
Please work through the assignments for a more complicated (but also more realistic) example.

# Example: "Decision Trees"

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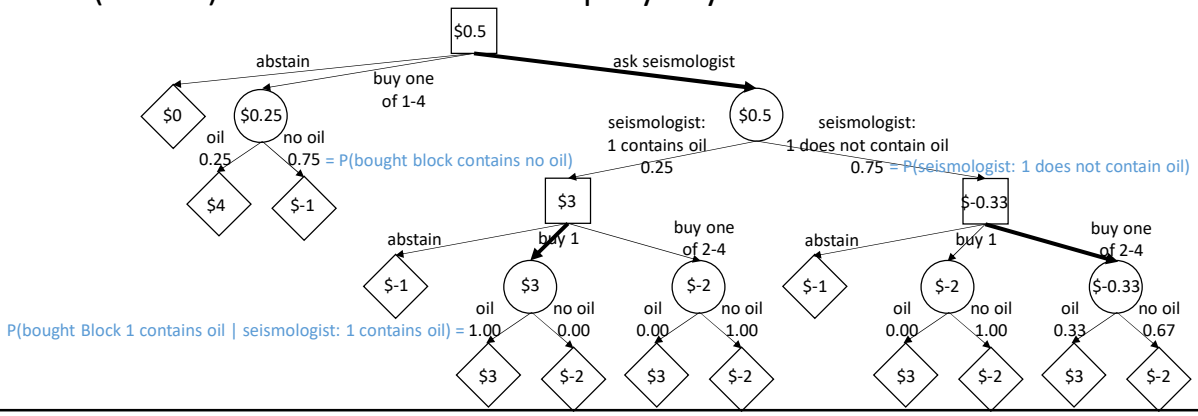


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# Example: "Decision Trees"

- Answer: Yes, for an expected profit of 0.5 (million) dollars.
- The plan is: Ask the seismologist. If the seismologist says that Block 1 contains oil, buy it. If the seismologist says that Block 1 does not contain oil, buy one of the other blocks.
- Note that this is a conditional plan, not a sequence of actions. If one should take the same action independently of what the seismologist says, then it does not make sense to pay the seismologist since one would ignore the obtained information. In general, plans are no longer sequences of actions for probabilistic planning.

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## Example: “Decision Trees”

- The seismologist now wants more money for the information. How much should the company pay the seismologist at most for their information?
- Answer: 1.25 (million) dollars, which makes the company indifferent between buying one of the blocks unseen or paying the seismologist for their information, for an expected profit of 0.25 (million) dollars in both cases.

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## Value of Information

- The value of information is the increase in profit (or reduction of cost) that results from having the information available if the information is free.
- The value of information is non-negative since having additional information available can never decrease the profit (since the information can just be ignored in the worst case). The value of information is strictly positive only if the information causes a change of plan.
- Information should be obtained only if the value of information is no less than the cost of the information.

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# Value of Information

- The value of information of the survey of Block 1 is 1.25 (million) dollars since it increases the profit from 0.25 (million) dollars (for buying one of the blocks unseen) to 1.5 (million) dollars (for buying Block 1 if the seismologist says that Block 1 contains oil and buying one of the other blocks if the seismologist says that Block 1 does not contain oil).
- Since information should be obtained only if the value of information is no less than the cost of the information, the company should pay the seismologist at most 1.25 (million) dollars (= the value of information) for their information.

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