Uninformed Search

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Russell and Norvig, 3rd Edition, Sections 10.2.1 and 3.3-3.4

These slides are new and can contain mistakes and typos.
Please report them to Sven (skoenig@usc.edu).

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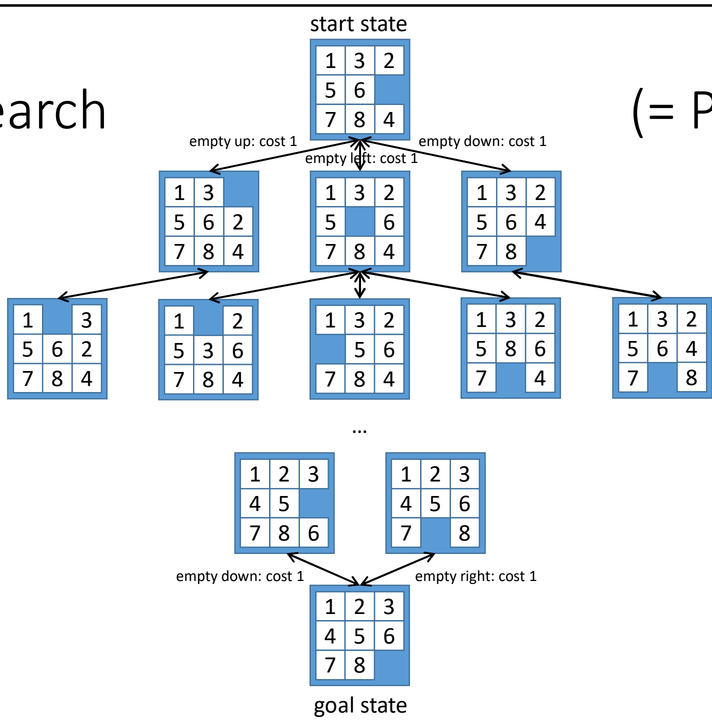
Search (= Path Planning)



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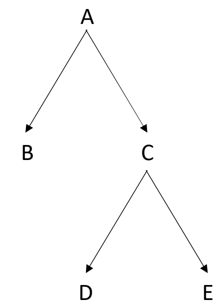
Search



- Terminology for trees
  - C is the parent node of D.
  - D and E are the child nodes of C.
  - A is the root node since it has no parent node.
  - B, D and E are the leaf nodes since they have no child nodes.
  - All leaf nodes form the fringe (or frontier).
  - The depth of node C is one since it is one edge traversal away from the root node.
  - The depth of the tree is two since the largest depth of any of its nodes is two.

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Skeleton of Search Algorithms

1. Start with a tree that contains only one node,
   labeled with the start state.
2. If there are no unexpanded fringe nodes, stop unsuccessfully.
3. Pick an unexpanded fringe node n. Let s(n) be the state it is labeled with.
4. If s(n) is a goal state, stop successfully
   and return the path from the root node to n in the tree
   (i.e. return the sequence of states that label the nodes on the path).
5. Expand n, that is, create a child node of n for each of the successor states of s(n), labeled with that successor state.
6. Go to 2.

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Skeleton of Search Algorithms

- The search algorithms differ only in how they select the unexpanded fringe node.
  - If no knowledge other than the current tree is available to guide the decision, then a search algorithm is called uninformed (or blind).
  - Otherwise, a search algorithm is called informed. If the knowledge consists of approximations of the goal distances of the states, the informed search algorithm is called heuristic.

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Breadth-First Search BFS (operator cost = one)

1. Start with a tree that contains only one node, labeled with the start state.
2. If there are no unexpanded fringe nodes, stop unsuccessfully.
3. Pick an unexpanded fringe node n with the smallest depth. Let s(n) be the state it is labeled with.
4. If s(n) is a goal state, stop successfully and return the path from the root node to n in the tree.
5. Expand n, that is, generate (= create) a child node of n for each of the successor states of s(n), labeled with that successor state.
6. Go to 2.

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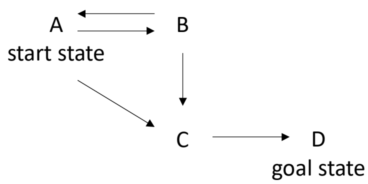
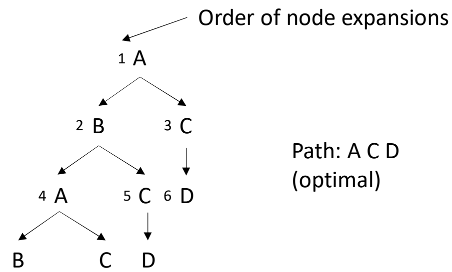
Breadth-First Search BFS (operator cost = one)

State space



Tree



We always break ties alphabetically in the following.

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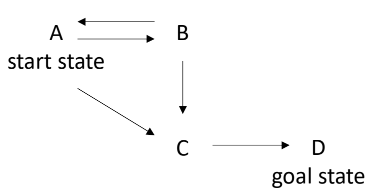
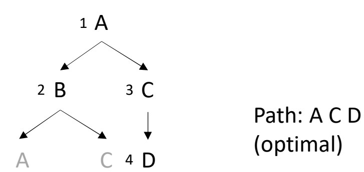
Breadth-First Search BFS (operator cost = one)

State space



Tree



- Pruning rule: do not expand a node if a node labeled with the same state has already been expanded. Thus, we can say that states get expanded rather than nodes.
- Optional termination rule: terminate once a node labeled with a goal state has been generated.

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Uniform-Cost Search (operator cost = positive)

1. Start with a tree that contains only one node, labeled with the start state.
2. If there are no unexpanded fringe nodes, stop unsuccessfully.
3. Pick an unexpanded fringe node n with the smallest cost g(n) from the root to n. Let s(n) be the state it is labeled with.
4. If s(n) is a goal state, stop successfully and return the path from the root node to n in the tree.
5. Expand n, that is, generate (= create) a child node of n for each of the successor states of s(n), labeled with that successor state.
6. Go to 2.

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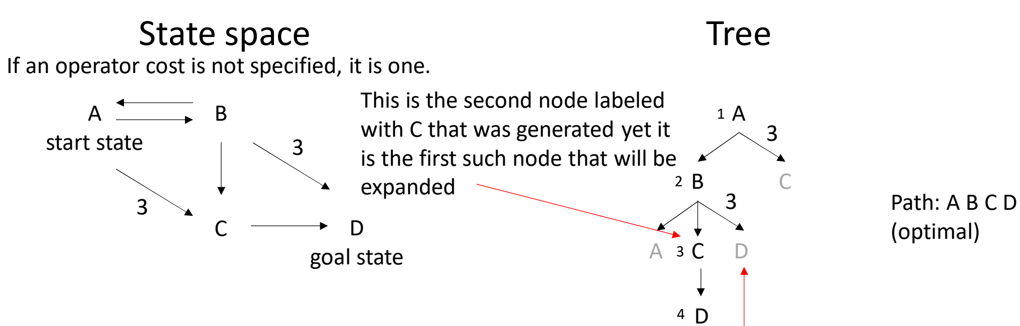
Uniform-Cost Search (operator cost = positive)



- Pruning rule: do not expand a node if a node labeled with the same state has already been expanded. Thus, we can say that states get expanded rather than nodes.
- ~~Termination rule: terminate once a node labeled with a goal state has been generated~~

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Properties

- Completeness
  - If an operator sequence exists that transforms the start state to a goal state, will the search algorithm report one?
- Time complexity
  - How many nodes are expanded at most?
- Space complexity
  - How many nodes need to be in memory at most at any point in time?
- Optimality
  - If an operator sequence exists that transforms the start state to a goal state, will the search algorithm report a cost-minimal one?

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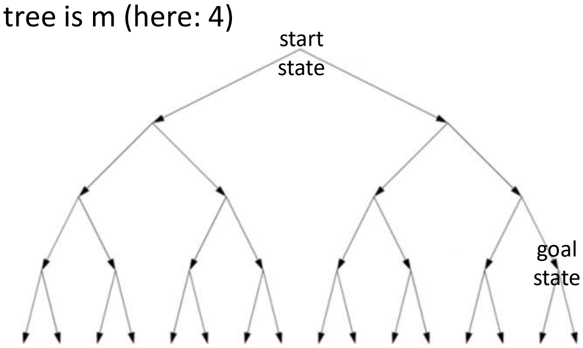
Properties

- We assume that the state space is a uniform tree when calculating the space and time complexity.
  - Branching factor is b (here: 2).
  - Depth of the goal state is d (here: 3).
  - Depth of the tree is m (here: 4)



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Properties

$b^0 + b^1 + \ldots + b^d = O(b^d)$

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes^a	Yes^{a,b}				
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$				
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$				
Optimal?	Yes^c	Yes				

Figure 3.21 Evaluation of tree-search strategies. b is the branching factor; d is the depth of the shallowest solution; m is the maximum depth of the search tree; l is the depth limit. Superscript caveats are as follows: ^{a} complete if b is finite; ^{b} complete if step costs $\geq \epsilon$ for positive ϵ; ^{c} optimal if step costs are all identical; ^{d} if both directions use breadth-first search.

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Breadth-First Search BFS (operator cost = one)

Depth	Nodes	Time	Memory
2	110	.11 milliseconds	107 kilobytes
4	11,110	11 milliseconds	10.6 megabytes
6	10^6	1.1 seconds	1 gigabyte
8	10^8	2 minutes	103 gigabytes
10	10^{10}	3 hours	10 terabytes
12	10^{12}	13 days	1 petabyte
14	10^{14}	3.5 years	99 petabytes
16	10^{16}	350 years	10 exabytes

Figure 3.13 Time and memory requirements for breadth-first search. The numbers shown assume branching factor $b = 10$; 1 million nodes/second; 1000 bytes/node.

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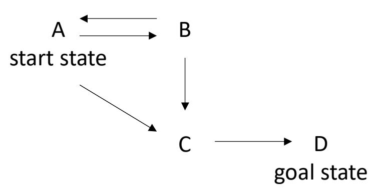
Depth-First Search DFS

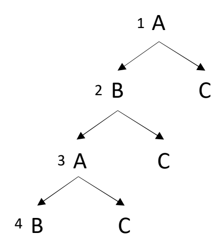
1. Start with a tree that contains only one node, labeled with the start state.
2. If there are no unexpanded fringe nodes, stop unsuccessfully.
3. Pick an unexpanded fringe node n with the largest depth. Let s(n) be the state it is labeled with.
4. If s(n) is a goal state, stop successfully and return the path from the root node to n in the tree.
5. Expand n, that is, generate (= create) a child node of n for each of the successor states of s(n), labeled with that successor state.
6. Go to 2.

Depth-First Search DFS (non-working)

State space



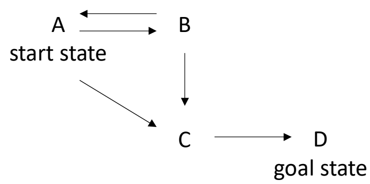
Tree



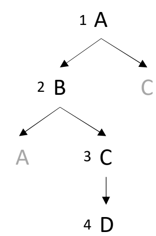
Note: In infinite state spaces (= with infinitely many states) DFS can go down a branch of the tree without a cycle, e.g. X1 – X2 – X3 – X4 – X5 – X6 – X7 - …

Depth-First Search DFS (memory inefficient)

State space



Tree



Path: A B C D
(non-optimal)

- Pruning rule: break cycles, for example, do not expand a node if a node labeled with the same state has already been expanded. Thus, we can say that states get expanded rather than nodes.
- Optional termination rule: terminate once a node labeled with a goal state has been generated.

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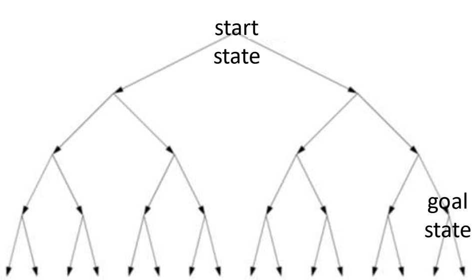
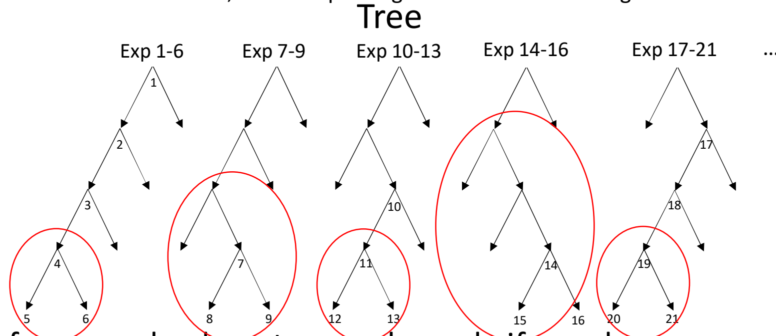
Depth-First Search DFS (memory efficient – use it)

Delete this from memory since it is no longer needed.
However, then the pruning rule needs to be changed.

State space



Tree



- Pruning rule: break cycles, for example, do not expand a node if a node labeled with the same state exists from the root node to the node in question.
- Optional termination rule: terminate once a node labeled with a goal state has been generated.

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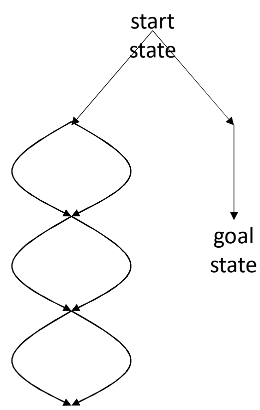
Depth-First Search DFS (memory efficient – use it)

- The new pruning rule is very weak.



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Properties

Yes, in finite state spaces.

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes[a] good	Yes[a,b]	No bad			
Time	$O(b^d)$ good	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$ bad			
Space	$O(b^d)$ bad	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$ good			
Optimal?	Yes[c] good	Yes	No bad			

Figure 3.21 Evaluation of tree-search strategies. b is the branching factor; d is the depth of the shallowest solution; m is the maximum depth of the search tree; l is the depth limit. Superscript caveats are as follows: [a] complete if b is finite; [b] complete if step costs $\geq \epsilon$ for positive ϵ; [c] optimal if step costs are all identical; [d] if both directions use breadth-first search.

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Iterative Deepening Search (operator cost = one)

- Combine the best properties of breadth-first and depth-first searches
- Implement a breadth-first search with a series of depth-first searches with increasing depth limits (that is, depth-first searches that assume that nodes whose depths are at least the depth limit have no children).

1. l := 0.
2. Perform a depth-first search with depth limit l.
3. If a node n labeled with a goal state was expanded,
   stop successfully and return the path from the root node to n in the tree.
4. If no node at depth l was expanded, stop unsuccessfully.
5. l := l+1.
6. Go to 2.

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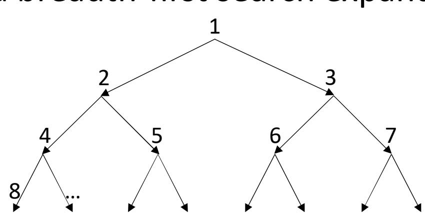
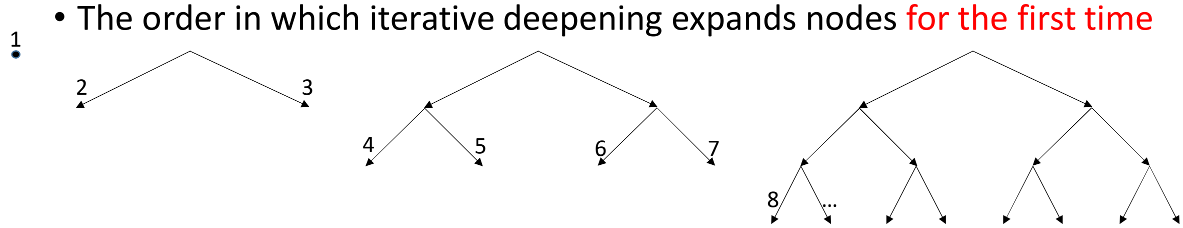
Iterative Deepening Search (operator cost = one)

- The order in which a breadth-first search expands nodes



- The order in which iterative deepening expands nodes for the first time



$O(b^0)$ + $O(b^1)$ + $O(b^2)$ + $O(b^3)$ + … + $O(b^d)$= $O(b^d)$

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Properties

If the branching factor is two, then about half of the expanded nodes are expanded for the first time during each depth-first search. This percentage is even larger for larger branching factors.

Yes, in finite state spaces.

- Iterative deepening incurs a time overhead over breadth-first search since it expands nodes multiple times. But the overhead is small and thus a small price to pay for the small space complexity.

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes[a]	Yes[a,b]	No	No	Yes[a] good	
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$ good	
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$ good	
Optimal?	Yes[c]	Yes	No	No	Yes[c] good	

Figure 3.21 Evaluation of tree-search strategies. b is the branching factor; d is the depth of the shallowest solution; m is the maximum depth of the search tree; l is the depth limit. Superscript caveats are as follows: [a] complete if b is finite; [b] complete if step costs $\geq \epsilon$ for positive ϵ; [c] optimal if step costs are all identical; [d] if both directions use breadth-first search.

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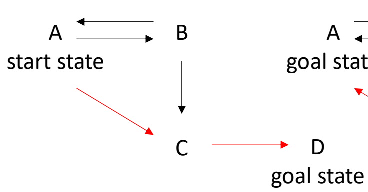
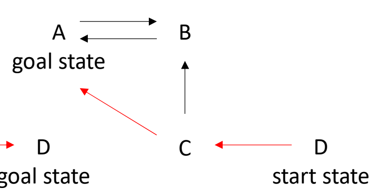
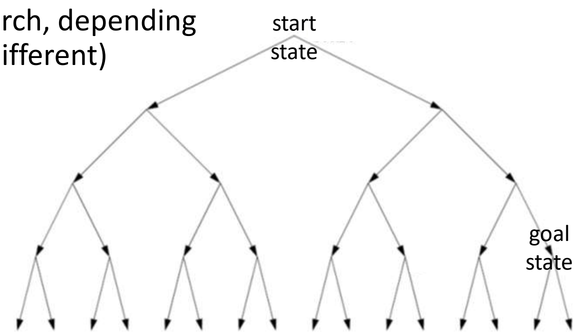
Bidirectional Search (here: operator cost = one)

- Properties of backward search (from goal state to start state)
  - Can be done in principle by reversing all operators and exchanging the start and goal states, which finds a path in reverse for the original state space. However, this might be difficult to do in practice (e.g. for STRIPS planning).
  - Can be faster or slower than forward search, depending on the branching factors (which can be different)







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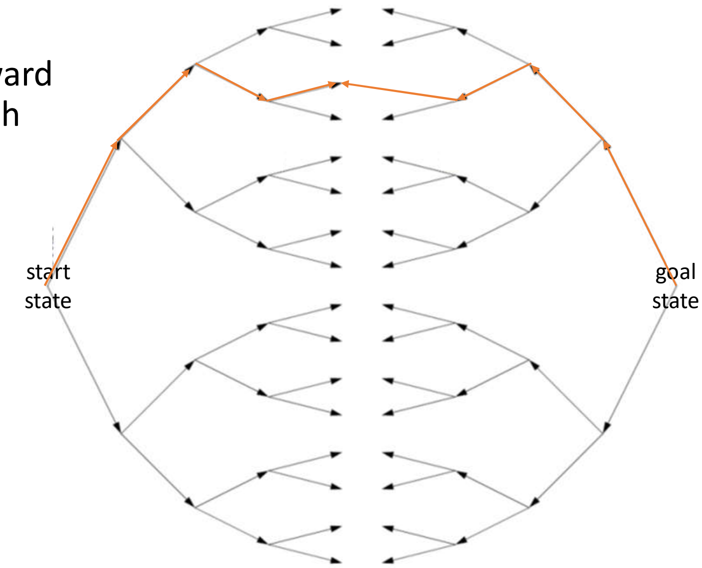
Bidirectional Search (here: operator cost = one)

- It helps if the forward or backward search is a breadth-first search (since one needs to test for an intersection of the trees)



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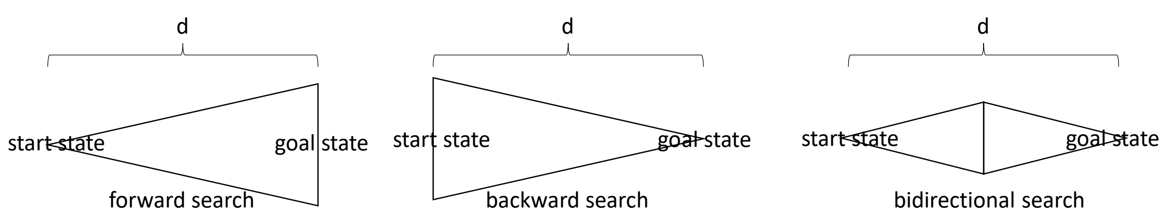
Bidirectional Search (here: operator cost = one)

- We assume that the state space is a uniform tree when calculating the space and time complexity.
  - Forward and backward branching factor is b
  - Depth of the goal state is d



$O(b^d)$ $O(b^d)$ $O(b^{d/2}) + O(b^{d/2}) = O(b^{d/2})$

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Properties

Yes, in finite state spaces.

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes[a]	Yes[a,b]	No	No	Yes[a]	Yes[a,d]
Time	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lfloor C^*/\epsilon \rfloor})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$
Optimal?	Yes[c]	Yes	No	No	Yes[c]	Yes[c,d]

Figure 3.21 Evaluation of tree-search strategies. b is the branching factor; d is the depth of the shallowest solution; m is the maximum depth of the search tree; l is the depth limit. Superscript caveats are as follows: [a] complete if b is finite; [b] complete if step costs $\geq \epsilon$ for positive ϵ; [c] optimal if step costs are all identical; [d] if both directions use breadth-first search.

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Uninformed Search

- Want to play around with uninformed search algorithms?
- Go here: http://aispace.org/search/

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