CS360 Homework 2– Solution

Resolution with Propositional Logic

1) Consider the following popular puzzle. When asked for the ages of her three children, Mrs. Baker says that Alice is her youngest child if Bill is not her youngest child, and that Alice is not her youngest child if Carl is not her youngest child. Write down a knowledge base that describes this riddle and the necessary background knowledge that only one of the three children can be her youngest child. Show with resolution that Bill is her youngest child.

Answer:
Let the propositions A, B and C denote that Mrs. Baker's youngest child is Alice, Bill and Carl, respectively. We have the following clauses for the background knowledge:

1 $A \vee B \vee C$ (One child has to be the youngest.)

2 $\neg A \vee \neg B$ (Alice and Bill cannot both be the youngest.)

3 $\neg A \vee \neg C$

4 $\neg B \vee \neg C$

The following clauses represent the information from Mrs. Baker:

5 $B \vee A$ (Alice is her youngest child if Bill is not her youngest child. That is, $\neg B \Rightarrow A$.)

6 $C \vee \neg A$ (Alice is not her youngest child if Carl is not her youngest child. That is, $\neg C \Rightarrow \neg A$.)

We want to show that Bill is the youngest child. Negating this, we get the following clause:

7 $\neg B$ (Assume that Bill is not the youngest child.)

We use resolution to derive the empty clause as follows:

8 (from 5,7) A

9 (from 3,6) $\neg A$

10 (from 8,9) $\perp$

2) Consider the following popular puzzle. A boy and a girl are talking. "I am a boy" said the child with black hair. "I am a girl" said the child with white hair. At least one of them is lying. Write down a knowledge base that describes this riddle. Show with resolution that both of them are lying.

Answer:

We use the following propositions:

- W_t: White haired child is telling the truth.
- W_b: White haired child is a boy.
- B_t: Black haired child is telling the truth.
- B_b: Black haired child is a boy.

We have the following clauses;

1 $B_b \vee W_b$ ("A boy and a girl are talking" means that at least one of them has to be a boy.)

2 $\neg B_b \vee \neg W_b$ (With the same logic, at least one of them has to be a girl.)

3 $\neg B_t \vee B_b$ (If the black haired child is telling the truth, it has to be a boy. That is, $B_t \Rightarrow B_b$.)

4 $B_t \vee \neg B_b$ (If the black haired child is lying, it has to be a girl. That is, $\neg B_t \Rightarrow \neg B_b$.)

5 $\neg W_t \vee \neg W_b$ (If the white haired child is telling the truth, it has to be a girl. That is, $W_t \Rightarrow \neg W_b$.)

6 $W_t \vee W_b$ (If the white haired child lying, it has to be a boy. That is, $\neg W_t \Rightarrow W_b$.)

7 $\neg B_t \vee \neg W_t$ (At least one of them is lying.)

We want to show that both of them are lying. That is, $\neg B_t \wedge \neg W_t$. Negating this, we get the following clause:

8 $B_t \vee W_t$ (Assume that at least one of them is telling the truth.)

We use resolution to derive the empty clause as follows:

9 (from 3,8) $B_b \vee W_t$

10 (from 5,9) $B_b \vee \neg W_b$

11 (from 1,10) B_b

12 (from 2,10) $\neg W_b$

13 (from 4,11) B_t

14 (from 6,12) W_t

15 (from 7,13) $\neg W_t$

16 (from 14,15) $\perp$

First Order Logic

3) Translate the following English sentences to first-order logic using the following predicates: Owns(x, y), Dog(x), Cat(x), Cute(x), and Scary(x). For example, Owns(x, y) means that object x owns object y:

(a) Joe has a cute dog.

Answer: $\exists x$ (Owns(Joe, x) $\wedge$ Dog(x) $\wedge$ Cute(x))

(b) All of Joe's dogs are cute.

Answer: $\forall x$ ((Owns(Joe, x) $\wedge$ Dog(x)) $\Rightarrow$ Cute(x))

(c) Unless Joe owns a dog, he is scary.

Answer: $\neg$($\exists x$ (Owns(Joe, x) $\wedge$ Dog(x))) $\Rightarrow$ Scary(Joe)

(d) Either Joe has at least one cat and at least one dog or he is scary (but not both at the same time).

Answer: ($\exists x$ (Owns(Joe, x) $\wedge$ Dog(x))) $\wedge$($\exists y$ (Owns(Joe, y) $\wedge$ Cat(y))) $\Leftrightarrow$ $\neg$ Scary(Joe).

(e) Not all dogs are both scary and cute.

Answer: $\exists x$ (Dog(x) $\wedge\neg$ (Scary(x) $\wedge$ Cute(x)))

4) Translate the following sentences in first-order logic to English. Apple(x) means that object x is an apple, Red(x) means that object s is red, Loves(x, y) means that person x loves person y:

(a) $\forall x$ (Apple(x) $\Rightarrow$Red(x))

Answer: All apples are red.

(b) $\forall x\ \exists y$ Loves(x, y)

Answer: Every person has some person he loves.

(c) $\exists y\ \forall x$ Loves(x, y)

Answer: There is a single person whom everybody loves.

5) Specify what a grandmother is, using the predicates IsGrandMotherOf, IsMotherOf and IsFatherOf. IsGrandMotherOf(x, y) means that person x is the grandmother of person y, IsMotherOf(x, y) means that person x is the mother of person y, and IsFatherOf(x, y) means that person x is the father of person y. Define additional predicates if needed.

Answer:

$\forall x, y$ (IsGrandMotherOf(x, y) $\Leftrightarrow$
$\exists z$ (IsMotherOf(x, z) $\wedge$ (IsMotherOf(z, y) $\vee$ IsFatherOf(z, y))))