

## Translating English Sentences to First Order Logic

In the following, it is important to remember the precedence of the operators, which are (from highest to lowest):  $\neg$  (NOT),  $\wedge$  (AND),  $\vee$  (OR),  $\Rightarrow$  (IMPLIES),  $\Leftrightarrow$  (EQUIV). Notice also that there are always several (equivalent) sentences in first-order logic that correspond to a given English sentence. We provide only one example.

All students are smart.

$\forall x (\text{Student}(x) \Rightarrow \text{Smart}(x))$

There exists a student.

$\exists x \text{Student}(x)$

There exists a smart student.

$\exists x (\text{Student}(x) \wedge \text{Smart}(x))$

Every student loves some student.

$\forall x (\text{Student}(x) \Rightarrow \exists y (\text{Student}(y) \wedge \text{Loves}(x,y)))$

Every student loves some other student.

$\forall x (\text{Student}(x) \Rightarrow \exists y (\text{Student}(y) \wedge \neg(x=y) \wedge \text{Loves}(x,y)))$

There is a student who is loved by every other student.

$\exists x (\text{Student}(x) \wedge \forall y (\text{Student}(y) \wedge \neg(x=y) \Rightarrow \text{Loves}(y,x)))$

Bill is a student.

$\text{Student}(\text{Bill})$

Bill takes either Analysis or Geometry (but not both).

$\text{Takes}(\text{Bill}, \text{Analysis}) \Leftrightarrow \neg \text{Takes}(\text{Bill}, \text{Geometry})$

Bill takes Analysis or Geometry (or both).

$\text{Takes}(\text{Bill}, \text{Analysis}) \vee \text{Takes}(\text{Bill}, \text{Geometry})$

Bill takes Analysis and Geometry.

$\text{Takes}(\text{Bill}, \text{Analysis}) \wedge \text{Takes}(\text{Bill}, \text{Geometry})$

Bill does not take Analysis.

$\neg \text{Takes}(\text{Bill}, \text{Analysis})$

No student loves Bill.

$\neg \exists x (\text{Student}(x) \wedge \text{Loves}(x, \text{Bill}))$

Bill has at least one sister.

$\exists x \text{SisterOf}(x, \text{Bill})$

Bill has no sister.

$\neg \exists x \text{ SisterOf}(x, \text{Bill})$

Bill has at most one sister.

$\forall x \forall y (\text{SisterOf}(x, \text{Bill}) \wedge \text{SisterOf}(y, \text{Bill}) \Rightarrow x=y)$

Bill has exactly one sister.

$\exists x (\text{SisterOf}(x, \text{Bill}) \wedge \forall y (\text{SisterOf}(y, \text{Bill}) \Rightarrow x=y))$

Bill has at least two sisters

$\exists x \exists y (\text{SisterOf}(x, \text{Bill}) \wedge (\text{SisterOf}(y, \text{Bill}) \wedge \neg(x=y)))$

Every student takes at least one course.

$\forall x (\text{Student}(x) \Rightarrow \exists y (\text{Course}(y) \wedge \text{Takes}(x, y)))$

Only one student failed History.

$\exists x (\text{Student}(x) \wedge \text{Failed}(x, \text{History}) \wedge \forall y (\text{Student}(y) \wedge \text{Failed}(y, \text{History}) \Rightarrow x=y))$

No student failed Chemistry, but at least one student failed History.

$\neg \exists x (\text{Student}(x) \wedge \text{Failed}(x, \text{Chemistry})) \wedge \exists x (\text{Student}(x) \wedge \text{Failed}(x, \text{History}))$

Every student who takes Analysis also takes Geometry.

$\forall x (\text{Student}(x) \wedge \text{Takes}(x, \text{Analysis}) \Rightarrow \text{Takes}(x, \text{Geometry}))$

No student can fool all the other students.

$\neg \exists x (\text{Student}(x) \wedge \forall y (\text{Student}(y) \wedge \neg(x=y) \Rightarrow \text{Fools}(x, y)))$